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A New Iterative Algorithm for Optimal Solution of Linear Programming Problems

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Abstract: This article deals with the minimization and maximization problem of LPP. The simplex method is the most popular and successful method for solving linear programs. The objective function of linear programming problem (LPP) involves in the minimization and maximization problem with the set of linear equalities and inequalities constraints. There are different method to solve LPP, such as simplex method, dual simplex method, Big-M method, Graphical method, Integer simplex method and two phase method. This paper leads to a technique to solve degeneracy occurring in simplex method and Dual simplex method in Linear programming Problems. We propose a new technique to choose the particular leaving and entering variable. This technique takes lesser time and less number of iteration than the existing method to obtain the optimal solution as compared to Dantzig pivot rule. The existing method always shows a required results proposed algorithm is better choice to avoid the confusions of taking arbitrary values to choose leaving variable, and hence the proposed algorithm is robust to solve degeneracy Linear Programming Problems.

Key words: Simplex Method, Dual Simplex Method, Occurrence of Degeneracy in Maximization and Minimization LPP

1. Introduction

To solve LPP, Simplex method is the popular and widely used method. Linear programming is a mathematical modelling technique useful for allocation of limited resources such as labour, materials, machines, time, cost etc. to several compelling activities. It is known that OR came into existence as a discipline during World War II to manage limited resources. Although a particular model and technique of OR can be traced back as early as in World War I when Thomas Edison (1914-1915) made an effort to use a tactical game board for solution to minimize shipping losses from enemy submarines. About the same time A.K. Erlang, a Danish engineer carried out experiments to study the fluctuations in demand for telephone facilities using automatic dialing equipment. The term OR was coined as a result of research on military operations during World War II. After that a group of specialists in Mathematics, Economics, Statistics, Engineering and other physical sciences were formed as special units within the armed forces to deal with strategic and tactical problems of various military problems. Following the success of this group OR became more useful. After World War II ended, efforts were made to apply OR approach to civilian problems related to business, industry, research and Development etc. Many operation researchers continued their research after World War; consequently many important advancements were made in OR techniques.

The Dual Simplex Method and other several methods have been developed, as variants of Simplex method to solve LPP problem by starting an infeasible solution to the primal. All these methods work in an iterative manner. They force the solution to become feasible as well as optimal simultaneously at some stages. To find optimal solution by an optimization technique is called Linear Programming. It is very important to be used in narrow fields of mathematics, Engineering, Business, Computer Science, Employment, Organization and personal Appointment.

The dual simplex method was developed by **Lemke**. Among all the methods it is widely used. It also works on thousands of constraints and variables also used. The dual simplex method plays most important role in the field of operations research of Mathematics, Engineering and also business. The dual simplex method maintains optimality and the successive iterations will work to clear infeasibility. When feasibility reaches the process terminates. Since the solution is both feasible and optimal. Therefore still now a day's lots of researchers used dual simplex method to solve LPP, but there are many confusions to resolve the issue of linear programming problems by a simple technique of linear programming problem. Therefore I am introducing a new iterative algorithm which resolves several confusions in the resolution of linear programming problems. When we convert a linear programming problem in the tabular form we have a confusion to take least negative value or most negative value for departing or entering variables. In this my research algorithm obtains same results on taking least negative value for leaving variable instead of most negative value .In different applied mathematics problems when once time takes greatest negative value and occurs same ratios then we will not be able to choose the accurate value at that we can achieve the correct resolution of the given problems. In such types of problems my iterative algorithm of Dual simplex method solves these problems to give a preference to least negative value. In case, same ratios appears we prefer dominating non basic variables. My technique is applicable for both minimization and maximization problems of LPP occurs degeneracy. In certain cases, in dual simplex method and simplex method there occur same ratio in solution column and in such cases the question arises for leaving variables. In these types of cases tie between leaving variable. The tie can be broken arbitrary, it is the degeneracy in dual simplex method, so the main aim of my research is to develop an algorithm to solve the degeneracy and have an optimal solution. [1] Develop a new technique to solve degeneracy in linear programming by simplex method to choose arbitrary values for leaving variable. [2] Introduce a pivot rule for maximization degeneracy problem of simplex method for linear programming. Improves the algorithm in the entering variable and leaving variable for the maximization degeneracy problems of LP. [3] Propose a new technique seven step process in LPP for the simplex, dual simplex, Big-M and two phase methods to get the solution with complexity reduction; the complexity reduction is done by eliminating the number of elementary row transformation operation in simplex tableau of identity matrix. [4] Suggests a new approach while solving two phase (Phase 1 and Phase 2) simplex method. Method attempts to replace more than one basic variable simultaneously. [5] Maximum improvement in the objective value function: from the set of basis-entering variables with positive reduced cost, the efficient basis-entering variable corresponds to an optimal improvement of the objective function. [6] In this research presents new and easy to use versions of primal and dual phase 1 processes which obviate the role of artificial variables and constraints by allowing negative variables into the basis. The new method is artificial free so, it also avoids stalling and saves degenerate pivots in many cases of linear programming problems. [8] In this approach collected and analyzed a number of linear programming problems that have been shown to cycle (not converge) when solved by Dantzig's original simplex algorithm. For these problems, some of the more popular linear programming solvers would find an optimal solution. [9] Deals with some forms of Two-Phase Unrevised Simplex Method (TPUSM). The results from an algebraic calculation are checked, using the TORA software, a computing software and of reference in linear programming. Solving Linear programming Problem (LPP) by TORA software, maximizing objective function and minimizing objective problem without any transformation.

In my paper, we have developed a new pivot rule called it (maximization $ minimization) degeneracy problem of simplex method and dual simplex method for LP. This technique is very suitable for such problems of degeneracy in (table A) which similar ratios arises. Due to degeneracy the problem is more difficult and takes large number of iteration to obtain the optimal solution. In such type of problem my developed algorithm plays a vital rule to get optimal solution. By small changing in leaving variable the problem resolve the issues of degeneracy and we get the optimal solution in small number of iteration. Finally we compare our result with dual simplex method and others methods solution.

Iteration	Basis	X_1	X_2	X_3	S_1	S_2	S_3	Solution
1	Z	a_1	a_2	a_3	0	0	0	0
	S_1	a_4	a_5	a_6	1	0	0	C
	S_2	a7	a_8	a9	0	1	0	C
	S_3	a_{10}	a_{11}	a_{12}	0	0	1	B

In this type of problem the degeneracy is very big issue of LPP. Due to degeneracy the problems will be more difficult and will take lot of time for its solution

2. Research Methodology

A standard form of L.P.P given as

$$\text{Minimize} \quad W = b_1y_1 + b_2y_2 + \dots + b_my_m$$

$$\text{Subject to} \quad a_{11}y_1 + a_{21}y_2 + \dots + a_{m1}y_m \geq c_1$$

$$a_{12}y_1 + a_{22}y_2 + \dots + a_{m2}y_m \geq c_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$a_{1n}y_1 + a_{2n}y_2 + \dots + a_{mn}y_m \geq c_n$$

$$y_i \geq 0, \text{for all } i = 1, 2, \dots, m.$$

Standard form of L.P Model

To initial step get all constraints to "≤" inequality and adding slack variables .Thus we get following standard form

Minimize $W - b_1y_1 - b_2y_2 - \dots -b_my_m \dots - 0s_1 - 0s_2 - \dots . 0s_m = 0$

Subject to $a_{11}y_1 + a_{21}y_2 + \dots + a_{1m}y_m + S_1 = - c_1$

$a_{21}y_1 + a_{22}y_2 + \dots + a_{m2}y_m + S_2 = - c_2$

$\dots\dots\dots\dots\dots\dots\dots\dots$

$a_{1n}y_1 + a_{2n}y_2 + \ldots + a_{mn}y_m + S_n = -c_n$
$y_1, y_2 \ldots. y_n, s_1, s_2, \ldots. s_n \geq 0$

3. Algorithm for Existing Method

Step1. **Standard Form**
Convert the constraint of type" ≥" to the constraint of type "≤" and adding slack variables.

Step2. **Leaving variable**
Select Leaving variable from basic variables having least negative value. If all the basic variables are non-negative then the problems has no feasible solution

Step3. **Entering variable**

(a) For entering variable takes the ratio test of the left hand side coefficients of Z-equation to the corresponding coefficient in the equation associated with departing variable.

(b) Choose smallest ratio in case of minimization and absolute value of ratio in case of maximization.

(c) **In case, same ratios appear we prefer dominating non basic variable.**

Step4. **New Solution**
After selecting entering variable and leaving variables, row operation are applied as usual to obtain the new solution.

Step5. **Optimality Test**
If $c_i \geq o$ then stop solution is an optimal, otherwise go to step two 2.

Number#1. **To obtained the pivot equation.**
Divide whole row where leaving variable appear by key element

Number#2. **To obtained all new equations including Z-equation**
New equation = old equation – (its entering column coefficient) × (New pivot equation)

4. Numerical Example # 1

Min $Z = 2x_1 + x_2$

Subject to
$3x_1 + x_2 \geq 3$
$4x_1 + 3x_2 \geq 6$
$x_1 + 2x_2 \leq 3$
$X_1, X_2 \geq 0$

Table# 1

Basis	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**Solution**
Z	**-2**	**-1**	**0**	**0**	**0**	**0**
S_1	**-3**	**-1**	**1**	**0**	**0**	**-3**
S_2	**-4**	**-3**	**0**	**1**	**0**	**-6**
S_3	**1**	**2**	**0**	**0**	**1**	**3**

In the minimization problems, we take value for leaving variable from the solution column. In the solution column -6 is the greatest negative number (Table #1) so S_2 is the leaving variable. To obtain the ratios, divide all the element of Z equation by S_2 row and get smallest ratio as 1/3 in the test ratio so non basic variable x_2 is an entering variable. After process we get table 2

Variable	X_1	X_2	S_1	S_2	S_3	solution
Z-Equ	-2	-1	0	0	0	0
S_2-Equ	-4	-3	0	1	0	6
Test Ratios	1/2	1/3	-	-	-	-

(Table# 2)

Basis	x_1	x_2	S1	S2	S3	Solution
Z	-2/3	0	0	-1/3	0	2
S1	-5/3	0	1	-1/3	0	-1
X_2	4/3	1	0	-1/3	0	2
S_2	-5/3	0	0	2/3	1	-1

In table#2 degeneracy occurs (S_1 and S_3) when solve by dual simplex method. To ovoid from the degeneracy and also takes less number of iteration than the others methods. We prefer to least negative number in solution column. Computation is given by

(Table#3)

Basis	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**Solution**
Z	**0**	**0**	**-2/5**	**-1/5**	**0**	**12/5**

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X_1	**1**	**0**	**-3/5**	**1/5**	**0**	**-1**
X_2	**0**	**1**	**4/5**	**-3/5**	**0**	**6/5**
S_3	**0**	**0**	**-1**	**1**	**1**	**0**

Results: Z = 12/5, X_1 = -1, X_3

Example#2.

Min $Z= 4x_1 +x_2$

Subject to $6x_1+ 2x_2 \geq 3$

$8x_1 + 6x_2 \geq 6$

$2x_1 -4x_2 \leq 3$

$X_1, X_2 \geq 0$

Results: Z = 12/7, X_1= 3/14 X_2= 6/7

Status: verified

Example# 3

Min $Z= 5x_1 +7x_2$

Subject to $2x_1+ 3x_2 \geq 42$

$3x_1 + 4x_2 \geq 60$

$x_1 + x_2 \geq 18$

$X_1, X_2 \geq 0$

Results: Z= 78, X_1 = 24 X_2 = -6

Status: verified

5. Results and Comperisions

Examples	**Method**	**No of Iteration**	**Results**	**Status**
Min $Z= 2x_1 +x_2$ Sub to $3x_1+ x_2 \geq 3$ $4x_1 + 3x_2 \geq 6$ $x_1 +2x_2 \leq 3$ $X_1, X_2 \geq 0$	My Technique Graphical Method Dual Simplex Method Two Phase Method Integers Simplex Method	**2** **3** **4** **4** **6**	Z = 12/7, X_1= 3/5 X_2= 6/5	**Verified**
Min $Z= 4x_1 +x_2$ Sub to $6x_1+ 2x_2 \geq 3$ $8x_1 + 6x_2 \geq 6$ $2x_1 - 4x_2 \leq 3$ $X_1, X_2 \geq 0$	My Technique Graphical Method Dual Simplex Method Two Phase Method Integers Simplex Method	**2** **3** **Fail** **Fail** **4**	Z = 12/7, X_1= 3/14 X_2= 6/7	**Verified**
Min $Z= 5x_1 +7x_2$ Sub to $2x_1+ 3x_2 \geq 42$ $3x_1 + 4x_2 \geq 60$ $x_1 + x_2 \geq 18$ $X_1, X_2 \geq 0$	My Technique Graphical Method Dual Simplex Method Two Phase Method Simplex Method Integers Method	**2** **3** **Fail** **5** **4**	Z= 78, X_1 = 24 X_2 = -6	**Verified**

6. Conclusion

In this proposed new iterative algorithm to find the accurate solution of minimization and maximization in Linear Programming problem in Dual Simplex method and Simplex method. The iterative algorithm sometimes involves less or at the most an equal number of iteration as compared to existing method .In the problem of LPP, when difficulty (Tie between leaving variable and entering variables) occurs, we need to take arbitrary elements as the leaving element and that produce difficulty, To remove that difficulty by this proposed algorithm to choose least negative value instead of most negative value and saves our time.

Mostly Engineering, Business, Economic, Computer Science and Educational problems are solved using such methods. By this new algorithm we have noticed that a proposed algorithm is faster and reduce number of iterations as compare to existing method and also give same optimal solution. Moreover, the proposed algorithm is equally efficient for simplex method and their types of degeneracy as well as cyclic linear programming problems.

7. References

1. A pivot Rule for Maximization Degeneracy problems of Simplex Method for linear programming problems by "R. K. MENGHWAR++ , F.SHAIKH" (vol.49 (004) 685-688(2017)
2. Development of new Technique to solve Degeneracy in linear programming by "R.

SHAIKH[++] A.A SHAIKH, S.F SHAH" (vol.49(3) 571-574(2017)

3. Effect of Seven Steps Approach on Simplex Method to Optimize the Mathematical Manipulation by "Mohammad Rashid Hussain, Mohammed Qayyum, Mohammad Equebal Hussain" (ISSN: 2277-3878, Volume-7 Issue-5, January 2019)
4. Application of Quick Simplex Method (A New Approach) On Two Phase Method by "N. V. Vaidya1* and N. N. Kasturiwale" (16 (1): 1-15, 2016, Article no.BJMCS.24440 ISSN: 2231-0851)
5. New optimal pivot rule for the simplex algorithm by "Jean Bosco Etoa", September 16, 2016.
6. New artificial –free Phase 1 Simplex Method by "Nasiruddin Khan, Syed Inayatullah*, Muhammad Imtiaz and FoziaHanif Khan" (August 2010 IJBAS Vol: 9 No: 10)
7. A Primal-Dual Simplex Algorithm for Solving Linear Programming Problems with Symmetric Trapezoidal Fuzzy Numbers by "Ali Ebrahimnejad" (2011, 2, 676-684 doi:10.4236/am.2011.26089)
8. Cycling in linear programming problems by Saul I. Gass[a];*, SasirekhaVinjamuri[b] (Computers & Operations Research 31 (2004) 303–311)
9. A New Modified Simplex Method to Solve Quadratic Fractional Programming Problem and Compared it to a Traditional Simplex Method by Using Pseudoaffinity of Quadratic Fractional Functions by "Nejmaddin A. Suleiman and Maher A. Nawkhass"(Vol. 7, 2013, no. 76, 3749 – 3764)
10. On the Two-Phase Unrevised Simplex Method by François Ndayiragije (vol. 7, issue 77, June 2018)
11. Alternative Approach to Wolfe's Modified Simplex Method for Quadratic Programming Problems by N. W. Khobragade, N. K. Lamba and P. G. Khot(Vol-2 No. 1 March 2012)

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